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jective method in mathematics" by B. Cosby, 451–455; "Tangent lines among the Greeks" by F. Cajori, 463–464; "Trigonometry without similar triangles" by J. A. Nyberg, 467–470; "A new ellipsograph?" by J. A. van Groos, 471–472—June: "The uses of algebra in study and reading" by E. L. Thorndike and Ella Woodyard, 514–522; "Graphical trisection of an angle" by E. D. Pickering, 548–549.

Science, new series, volume 55, May 5, 1922: "The Edward C. Pickering memorial" by S. A. Mitchell, 467–469 [A fund for the Association of Variable Star Observers to be devoted to variable star research]—May 12: "Mathematical publications," 508–509; "The Eliakim Hastings Moore Fund" by A. Dresden, 510—May 26; "The Einstein equations of the solar field from the Newtonian point of view" by L. P. Eisenhart, 570–572.

AMERICAN DOCTORAL DISSERTATIONS.

J. D. Barter, "The homogeneous vector function and determinants of the P-th class," *University of California Publications in Mathematics*, vol. 1, 1920, pp. 321–343. (University of California, 1917.)

Teresa Cohen, "Investigations on the plane quartic." Pp. 191–211. [Reprinted from *American Journal of Mathematics*, vol. 41, 1919.] (Johns Hopkins University, 1918.)

H. D. Frary, "The Green's function for a plane contour." Pp. 11–25. [Reprinted from *American Journal of Mathematics*, vol. 42, 1920.] (University of Illinois, 1918.)

K. W. Lamson, "A general implicit function theorem, with an application to problems of relative minima." Pp. 243–256. [Reprinted from *American Journal of Mathematics*, vol. 42, 1920.] (University of Chicago, 1917.)

Flora E. Le Stourgeon, "Minima of functions of lines." Pp. 357–383. [Reprinted from *Transactions of the American Mathematical Society*, vol. 21, 1920.] (University of Chicago, 1917.)

Wayne Sensenig, "Concerning the invariant theory of involutions of conics." Pp. 111–122. [Reprinted from *American Journal of Mathematics*, vol. 41, 1919.] (University of Pennsylvania, 1919.)

G. W. Smith, "Nilpotent algebras generated by two units, i and j, such that i^2 is not an independent unit." Pp. 143–164. [Reprinted from *American Journal of Mathematics*, vol. 41, 1919.] (University of Illinois, 1917.)

J. S. Taylor, "A set of five postulates for Boolean algebras in terms of the operation exception,'" *University of California Publications in Mathematics*, vol. 1, 1920, pp. 241–248. (University of California, 1918.)

L. E. Wear, "On self-dual plane curves of the fourth order." Pp. 97–118. [Reprinted from *American Journal of Mathematics*, vol. 42, 1920.] (Johns Hopkins University, 1913.)

PROBLEMS AND SOLUTIONS.

Edited by B. F. Finkel, Otto Dunkel, and H. P. Manning.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N. B. Problems containing results believed to be new, or extensions of old results, are especially sought. The editorial work would be greatly facilitated if, on sending in problems, proposers would also enclose any solutions or information that will assist the editors in checking the statements. In general, problems in well-known text-books, or results found in readily accessible sources, will not be proposed as problems for solution in the Monthly. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

2977. Proposed by FLORENCE P. LEWIS, Goucher College.

A point moves in such a way that its polars with respect to two given conics intersect at right angles. Prove that the locus of this intersection is a rational quartic curve through the circular points, and find its double points.

2978. Proposed by C. N. SCHMALL, New York City.

Given the equation of the general cubic $f(x, y) \equiv ax^3 + 3bx^2y + 3cxy^2 + dy^3 + a_1x^2 + 2b_1xy + c_1y^2 + a_2x + b_2y + c_2 = 0$, show that the three asymptotes of the curve will be concurrent if

$$\begin{vmatrix} a & b & a_1 \\ b & c & b_1 \\ c & d & c_1 \end{vmatrix} = 0.$$

2979. Proposed by V. M. SPUNAR, Chicago, Ill.

If a tangent be drawn from a variable point of an ellipse of length equal to n times the semi-conjugate diameter at the point, the locus of its extremity will be a concentric ellipse with semi-axes equal to $a\sqrt{n^2 + 1}$, $b\sqrt{n^2 + 1}$.

2980. Proposed by J. ROSENBAUM, Milford, Conn.

Locate a point such that the sum of its distances from the vertices of a given polygon shall be a minimum.

SOLUTIONS.

2876 [1921, 89]. Proposed by H. S. UHLER, Yale University.

Let a, d, n, and r denote respectively the refracting angle of a prism, the total deviation of the ray (in a principal plane) produced by the prism, the relative index of refraction of the material of the prism with respect to the single surrounding medium, and the angle of refraction at the first or incidence face of the prism.

(a) Deduce the (new) formula given below. (b) Use this formula to show (without the calculus) that the deviation has the least value when the ray is symmetrically situated with respect to the two refracting faces. (c) Show that, for minimum deviation, the formula reduces to the classical laboratory expression for determining the index of refraction.

$$\frac{\sin^2 \frac{1}{2}(a + d)}{\sin^2 \frac{1}{2}a} = n^2 + \frac{\sin \frac{1}{2}d \sin (a+\frac{1}{2}d) \sin^2 \frac{1}{2}(a+d) \sin^2 (r-\frac{1}{2}a)}{[\sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a+d) \cos^2 (r-\frac{1}{2}a)+\cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a+d) \sin^2 (r-\frac{1}{2}a)] \sin^2 \frac{1}{2}a}.$$

SOLUTION BY THE PROPOSER.[1]

Let i and i' denote, respectively, the angles which the segments of the ray outside the prism make with the normals at the points of incidence and emergence. Let r and r' be the corresponding angles inside the prism.

(a) From the geometry of the obvious diagram

$$r + r' = a$$

and

$$i + i' = a + d.$$

Put $r = \frac{1}{2}a + x$, and therefore $r' = \frac{1}{2}a - x$. Then

$$\sin i = n \sin (\tfrac{1}{2}a + x)$$

and

$$\sin i' = n \sin (\tfrac{1}{2}a - x).$$

Adding and subtracting, we have

$$2 \sin \tfrac{1}{2}(a + d) \cos \tfrac{1}{2}(i - i') = 2n \sin \tfrac{1}{2}a \cos x,$$
$$2 \cos \tfrac{1}{2}(a + d) \sin \tfrac{1}{2}(i - i') = 2n \cos \tfrac{1}{2}a \sin x;$$

whence

$$n^2 = \frac{\sin^2 \frac{1}{2}(a + d) \cos^2 \frac{1}{2}(a + d)}{\sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a + d) \cos^2 x + \cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a + d) \sin^2 x}$$

or

$$\begin{aligned} \frac{\sin^2 \frac{1}{2}(a + d)}{\sin^2 \frac{1}{2}a} &= n^2 + \frac{[\cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a + d) - \sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a + d)] \sin^2 \frac{1}{2}(a + d) \sin^2 x}{[\sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a + d) \cos^2 x + \cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a + d) \sin^2 x] \sin^2 \frac{1}{2}a} \\ &= n^2 + \frac{\sin \frac{1}{2}d \sin (a + \frac{1}{2}d) \sin^2 \frac{1}{2}(a + d) \sin^2 x}{[\sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a + d) \cos^2 x + \cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a + d) \sin^2 x] \sin^2 \frac{1}{2}a}, \end{aligned}$$

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[1] See *1921*, 1. The proof and the formulæ given in this solution apply directly to the more general case of the deviation of the projected ray; that is, to D' of the paper referred to.